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Infrared radiation associated with protostars

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Various stages in the collapse of an interstellar gas cloud into a cluster of protostars are discussed. The residual gas of the cloud which does not go into the protostars should form an opaque envelope which will thermalize the energy output of the protostars and be detectable as a high-luminosity infrared object. The infrared object in Orion newly discovered by Low is interpreted to be one of these objects.

Several years ago (Cameron 1962) I discussed the conditions necessary for the initiation of the collapse of an interstellar gas cloud with the subsequent formation of stars. Recent observations by Low (1967) of an unusual infrared object have prompted a more detailed consideration of the collapse phase, which is presented below.

Interstellar clouds typically have masses of *ca.* $10^3 M_\odot$. For our present purposes it is sufficiently accurate to consider them to be uniform spheres and to estimate the conditions at the onset of collapse from the virial theorem condition

$$\Omega + 2U + 2K_t + 2K_r = \mathrm{d}^2 I/\mathrm{d}t^2 < 0, \quad (1)$$

where Ω is the gravitational potential energy, U the internal thermal energy, K_t the internal turbulent energy, K_r the rotational energy, and I the moment of inertia. It was shown in the previous paper (Cameron 1962) that collapse instability occurs when the hydrogen number density $n_{\mathrm{H}} \sim 10^3\,\mathrm{cm}^{-3}$. Under these conditions U and K_t are comparable, but K_r is small on the assumption that the angular velocity of the cloud $\omega = 10^{-15}$ rad/s corresponding to synchronous rotation about the centre of the galaxy. This density is at the upper limit of the densities observed in interstellar clouds, and it can be produced only if the gas is compressed along the lines of force of the interstellar magnetic field, whereupon the magnetic energy associated with the cloud is negligibly small. Parker (1966) has presented a mechanism in which interstellar clouds are naturally produced by gas motions along the lines of force. An external pressure, produced for example by a surrounding H II region, can assist in producing collapse but does not alter the order of magnitude of the physical parameters involved.

The temperature of the cloud is established by the approximate equality of heating and cooling processes. The cooling processes all depend upon two-body collisions and hence the cooling rate per unit mass is proportional to the density. During the collapse the gas is heated by compression, but the cooling becomes progressively more effective, and hence the collapse takes place under roughly isothermal conditions. Recent hydrodynamic calculations by W. D. Arnett show that the temperature probably decreases in the denser parts of the collapsing cloud. Hence the internal thermal energy does not increase during the collapse, whereas the magnitude of the gravitational potential energy becomes progressively larger.

Since the cloud is brought to the verge of collapse by a dynamic compression process, it is likely to be highly turbulent. There will be a spectrum of eddy sizes up to appreciable fractions of the dimensions of the cloud. However, supersonic turbulent velocities will be rapidly dissipated on all scales by shock waves. Subsonic turbulent velocities can be dissipated through generation of acoustic waves, but this process has an efficiency proportional to the fifth power of the Mach number (Lighthill 1952; Proudman 1952), and hence such dissipation can be neglected. Hence the typical turbulent velocity can be set equal to sound speed:

$$v_t \sim v_s = \left[\frac{\gamma N_0\, kT}{\mu}\right]^{\frac{1}{2}}, \tag{2}$$

where γ is the ratio of specific heats, N_0 is the Avogadro number, k is the Boltzmann constant, and μ is the mean molecular weight. These considerations show that K_t also remains approximately constant during the collapse.

If the interstellar cloud were to undergo a complete collapse without fragmentation, then it would ultimately form a flattened rotating disk. We can estimate the dimensions of the disk in equilibrium from the Virial Theorem expression of equation (1), ignoring the U and K_t terms:

$$\Omega + 2K_r = 0. \tag{3}$$

Here we can make the rough estimates

$$\Omega = -\tfrac{3}{5}GM^2/R, \tag{4}$$

where G is the gravitational constant, M the mass of the cloud, and R the radius of the cloud, and

$$K_r = \tfrac{1}{5}MR^2\omega^2, \tag{5}$$

where ω is the angular velocity. The final angular velocity can be estimated from conservation of angular momentum,

$$R^2\omega = \text{constant}. \tag{6}$$

For the initial radius we obtain from (1), (2) and (4),

$$U = \tfrac{3}{2}N_0 kTM/\mu \tag{7}$$

and

$$K_t = \tfrac{1}{2}Mv_t^2 = \gamma N_0 kTM/\mu \tag{8}$$

the expression

$$R = \frac{3}{5(3+\gamma)}\,\frac{GM\mu}{N_0 kT}. \tag{9}$$

With $M = 10^3 M_\odot$, $\mu = 1{\cdot}2$, $\gamma = \frac{5}{3}$, and $T = 50\ ^\circ\mathrm{K}$, we obtain $R_0 = 5\times 10^{18}$ cm as the initial radius of the cloud. As mentioned above, we take $\omega_0 = 10^{-15}$ rad/s as the initial angular velocity. The final radius is then

$$R_f = \tfrac{2}{3}(R_0^2\omega_0)^2/GM, \tag{10}$$

which gives $R_f = 3\times 10^{15}$ cm $=$ 200 astronomical units.

With fragmentation, the individual fragments will probably pass within a distance of this order, or somewhat larger, of the centre of the mass distribution. One of the features of the hydrodynamic calculation of W. D. Arnett is that density fluctuations lead to rapid collapse of the denser material. Such density fluctuations should be formed continually as a result of the shock dissipation of turbulence. Empirically, we expect that the ultimate fragment size will be of order one solar mass, but these fragments will result from a series of mass divisions during the contraction. We may estimate the threshold condition for condensations of any mass through use of equation (9). Let us assume that hydrogen atoms have had time to recombine into molecules on grain surfaces, so $\mu = 2{\cdot}3$. Again we take $T = 50$ °K, and obtain $R = 10^{16}$ cm for a fragment of one solar mass. At the time this fragmentation occurs the radius of the cloud would be 10^{17} cm.

It is of some interest to determine the time scales associated with the collapse. The free-fall time from a position of initial rest is

$$t_{\text{f.f.}} = \frac{\pi}{2}\left[\frac{R^3}{2GM}\right]^{\frac{1}{2}}. \tag{11}$$

With $R = 5\times10^{18}$ cm and $M = 2\times10^{36}$ g, $t_{\text{f.f.}} = 3{\cdot}4\times10^{13}$ s $= 1{\cdot}1\times10^{6}$ y. The free-fall velocity is

$$v_{\text{f.f.}} = \left[2GM\left(\frac{1}{R}-\frac{1}{R_0}\right)\right]^{\frac{1}{2}}, \tag{12}$$

where R_0 is the initial radial distance. Taking $M = 2\times10^{36}$ g, $R_0 = 5\times10^{18}$ cm, and $R = 10^{17}$ cm, we obtain $v_{\text{f.f.}} = 1{\cdot}6\times10^{6}$ cm/s. This indicates that the fragments will be in the vicinity of the mass centre after an additional *ca.* 2000 y. The danger of collisions among the fragments will be greatly minimized, however, as a result of the velocity dispersion associated with the density fluctuations giving rise to the series of mass divisions. The same density fluctuations will cause the individual fragments to complete their collapse before the fragments themselves reach the vicinity of the mass centre. This also helps to minimize the chances of collision.

After passing near the centre of mass of the cloud, the fragments will recede nearly to the original radius of the cloud. The times involved will be the same as the free-fall times: 2000 years to reach 10^{17} cm and $1{\cdot}1\times10^{6}$ years to reach 5×10^{18} cm for a fragment near the edge of the original cloud.

Let us now consider the character of the collapsed fragments. As we see from (6), the angular momentum associated with material at the edge of a fragment is proportional to $R^2\omega$. The contribution from the initial angular velocity of the cloud for the edge of a fragment of solar mass is $R^2\omega = (5\times10^{17})^2\times10^{-15} = 2{\cdot}5\times10^{20}$. At the time of fragmentation there may be eddies present with dimensions comparable to the fragment dimension. These will have turbulent velocity v_t with a component transverse to the radial direction of $\sqrt{\frac{2}{3}}v_t$. If we assume $T = 50$ °K, $v_t = 8\times10^{4}$ cm/s. Hence

$$R^2\omega = \sqrt{\tfrac{2}{3}}\,v_t R. \tag{13}$$

For $R = 10^{16}$ cm, $R^2\omega = 6\times10^{20}$. This maximum component of the turbulent angular momentum will add vectorially in a random fashion to the spin component given above. Hence values of $R^2\omega$ in the range $(3\text{–}8)\times10^{20}$ can be expected for the fragments.

In a previous publication (Cameron 1963) it was shown that the collapse of a cloud fragment should lead to the formation of a gas disk, probably with a pronounced central concentration to the spin axis, but too dispersed to allow the presence of a central body in spherical hydrostatic equilibrium. The radii of these disks are proportional to the square of the angular momentum of the fragment. With the spin angular momentum only taken into account, it was found (Cameron 1963) that a fragment of one solar mass would form an axially condensed disk with a radius of about 30 astronomical units. The preceding discussion indicates that this figure may be increased by a moderate factor. The resulting disks will have a distribution of spin axes which, while not random, is spherically distributed. It would probably be difficult, with present observational techniques, to distinguish this distribution from an isotropic one.

Because the disks are formed as the end-products of a dynamical collapse process, it must be expected that they will initially be highly turbulent. For an axially condensed disk the angular velocity for Kepler circular motion falls off inversely as the distance from the axis (Cameron 1963). Any similar type of mass distribution will likewise have a gradient in the angular velocity. Turbulent viscosity tends to eliminate such angular velocity gradients, and hence it tends to redistribute the mass in the disk, with most of the mass flowing toward the centre, but with the dimensions of the disk increasing to conserve total angular momentum. Ter Haar (1950) has estimated that the disk dissipation time due to turbulence is of order 1000 y. It is important to note that this dissipation time, while obviously highly uncertain, is comparable to the residence time of the fragments inside a radius of 10^{17} cm.

The compression of the gas in the formation of the disks leads to at least partial ionization of the hydrogen (Cameron 1962), and hence to temperatures of *ca.* 10^4 °K. The disks can cool by radiation to the range 500 to 1000 °K in just a few years (Cameron 1962), and hence initially they are sources of high luminosity.

It is evident that the dissipation of the mass in the disk will lead to a body in spherical hydrostatic equilibrium at the centre of the disk. The mass of this body must progressively increase as the dissipation proceeds. Two possible cases arise:

(1) The body is at all times a pre-main-sequence star in the Hayashi contraction phase. In this case its radius during the disk dissipation phase will tend to lie in the vicinity of 10^{12} cm.

(2) The disk cools rapidly enough so that the body forms as a relatively cold degenerate object. Mass addition to the surface leads to compressional heating. At some point the combination of high temperature and high density will ignite nuclear reactions, leading to an expansion of the body to the Hayashi contraction phase. During the degenerate phase the radius will be of order 10^9 cm.

In the Hayashi phase the star is fully convective, and hence it is approximately a polytrope of index 1·5. Hence the gravitational potential energy becomes $-\frac{6}{7}GM^2/R$. The same expression is also approximately correct for a star in the degenerate phase. Hence during the dissipation of the disk an energy of order $\frac{3}{7}GM^2/R$ will be radiated away. For final radii in the range 10^9–10^{12} cm, the total energy release thus lies in the range 10^{47}–10^{50} erg. For subsequent calculations we shall adopt the conservative value of 10^{47} erg.

We have noted that density fluctuations lead some parts of the interstellar cloud to collapse faster than other parts. In particular, the calculations of W. D. Arnett indicate that the mass near the edge of the cloud is likely to lag behind the collapse of the central portion. Let us assume for convenience that the condensation and fragmentation will produce 500 protostars of one solar mass each.

It then follows that the compact cluster of protostars will radiate at least 5×10^{49} erg in 1000 y, or at least 2×10^{39} erg/s.

Now consider the character of the residual 500 solar masses of gas which has not condensed into protostars. During the collapse this gas will remain at low temperature. The opacity of the material is almost entirely due to interstellar grains, and as long as the cloud has nearly its original dimensions the gas will be optically thin in the infrared.

When the radius of the residual gas cloud decreases to dimensions comparable to those at the final fragmentation stage, the gas becomes optically thick in the infrared. The low-temperature opacity of the interstellar grains has been discussed by Gaustad (1963). For a temperature of 100 °K he finds the Rosseland mean opacity to be $K\rho \sim 3{\cdot}9\times10^{-24}n\ \text{cm}^{-1}$, where n is the number density of atoms. This is equivalent to $K = 2\ \text{cm}^2/\text{g}$.

The optical depth at the centre of the residual gas is $\tau = K\rho R$. Hence

$$\tau = 3KM/4\pi R^2. \tag{14}$$

For $M = 10^{36}$ g and $R = 10^{17}$ cm, $\tau = 50$. The time required for a photon to escape from the gas is of order $\tau R/c$, which is about 7 y in this example. The absorption efficiency for the infrared radiation is of order 0·01, so that the optical depth for visible light at the centre will be of order 5000.

This rapid rate of energy loss greatly exceeds the rate at which the gas cloud could generate energy through its own contraction. It was for this reason that Gaustad (1963) pointed out that a gas cloud with this range of dimensions could not be in hydrostatic equilibrium. However, the gas cloud under consideration here has a cluster of strong energy sources in its interior. Consequently the question of equilibrium requires additional consideration.

From virial theorem considerations we can write approximately for equilibrium

$$\frac{3}{5}\frac{GM^2}{R} = 3kT\frac{N_0}{\mu}M_g + \tfrac{4}{3}\pi R^3 aT^4, \tag{15}$$

or

$$\tfrac{4}{3}\pi a(RT)^4 + \frac{3kN_0M_g}{\mu}RT - \tfrac{3}{5}GM^2 = 0. \tag{15a}$$

Here M is the total mass of the gas cloud, including condensations, and M_g is the mass of the gas in the cloud. We note that the equilibrium condition leads to a polynomial in RT. For our representative conditions we have $RT = 4{\cdot}7\times10^{19}$.

The luminosity of the gas cloud is approximately the total radiative energy content divided by the photon escape time:

$$L = \frac{4\pi}{3}\frac{R^3aT^4}{\tau R/c}$$
$$= \frac{16\pi^2}{9}\frac{ac}{KM_g}(RT)^4 \tag{16}$$

from equation (14). The opacity K is approximately inversely proportional to the wavelength of the light near the peak of the Planck spectrum (Gaustad 1963). For temperatures below 200 °K, the opacity of ice grains is therefore roughly $K = 2(T/100)\ \text{cm}^2/\text{g}$. Above 200 °K the ices evaporate, and the opacity is probably reduced by about a factor 20 (Gaustad 1963). In this region we can write $K = 2(T/2000)\ \text{cm}^2/\text{g}$. The luminosity thus becomes

$$L = \frac{8\pi^2}{9}\frac{ac}{M_g}(RT)^4\left(\frac{b}{T}\right), \tag{16a}$$

where b is 100 or 2000 as appropriate.

For our representative parameters we obtain $L = 1{\cdot}0 \times 10^{40}(b/T)$ erg/s. It is clear that the luminosity will attain a minimum value of 5×10^{39} erg/s at $T = 200$ °K.

In a star there is a self-adjusting regulatory mechanism which matches the energy generation to the luminosity through modification of the internal structure. However, in the present case the luminosity is provided by internal energy sources independent of the cloud structure. Hence the ability of cloud to reach hydrostatic equilibrium depends upon whether it can readjust its structure to conform to the internal energy sources. It is evident that this will be impossible unless the internal energy sources generate more than the necessary minimum luminosity requirements. In our example the expected luminosity is of the same order of magnitude as the minimum required for hydrostatic stability, so definite predictions cannot be made.

Let us now consider the new object found by Low (1967) in Orion. This object has a radius of 10^4 astronomical units, or $1{\cdot}5 \times 10^{17}$ cm. It has a surface temperature of 100 °K, and a corresponding luminosity of $1{\cdot}6 \times 10^{39}$ erg/s. This luminosity is comparable to the minimum value of the possible range of luminosities discussed above that could characterize a cluster of protostars. This suggests that the process of dissipation of a protostar disk does not involve a degenerate stellar stage.

The luminosity is somewhat less than the minimum required for hydrostatic equilibrium in our representative model. However, such parameters as mass and opacity are certainly not well enough known to reach any definite conclusions about this point. However, it should be noted that if the gas cloud is still essentially in free-fall collapse, its remaining lifetime would be only about 2000 y. The lifetime of the energy sources in the disk dissipation stage is only of order 1000 y. Also, in a time of the same order the energy sources should have passed through the minimum distance to the cloud centre and will emerge from the cloud. Hence the question of hydrostatic stability is not very significant, and the principal effect of the cloud will be to thermalize the radiation from the protostar cluster.

The study of the emerging protostars will give important information about the details of the disk dissipation processes. In our earlier discussion I assumed that turbulent viscosity would lead to mass flow toward the axis and to angular momentum transport away from the axis. There are two general mechanisms which could govern the energy dissipation that takes place:

(1) As mass concentration towards the axis takes place, the released gravitational potential energy goes into local heat. Hence most of the energy release occurs close to 10^{12} cm. If the protostar luminosity is about 3×10^{36} erg/s, then most of the radiation will be characterized by a temperature of *ca.* 8000 °K.

(2) As mass concentration towards the axis takes place, the released gravitational potential energy goes into turbulence, and an outward transport of turbulent energy accompanies the outward transport of angular momentum. In this case the energy will be dissipated into heat throughout the disk. For example, if the protostar radiates 3×10^{36} erg/s from a disk of radius 30 astronomical units, the characteristic temperature is 450 °K.

If the disk dissipation is completed, then the protostar should be a Hayashi-phase object with a temperature of *ca.* 4000 °K (Ezer & Cameron 1967).

A very cool stellar object in Orion was discovered by Becklin & Neugebauer (1967). Low (1967) has found this object to be near the edge of the 100 °K cloud and to have a well defined blackbody spectrum corresponding to a temperature of 500 °K. This may be a protostar emerging from the cloud or lagging behind the collapse of the cloud. More information about it will be needed to distinguish between the models just discussed. The object is consistent with turbulent transport of energy throughout the disk. However, if it is still embedded near the edge of the cloud, then it could have a temperature of several thousand degrees. This visible radiation would be readily absorbed by the cloud. If the local gas temperature in the cloud rises to 500 °K, the ices evaporate from the interstellar grains and the opacity is lowered (Gaustad 1963). This could allow thermalization of the radiation in a local region of low opacity, which we would then see through a thin high opacity layer at the edge of the 100 °K cloud. More observations will be needed to clarify the physical conditions involved.

From this discussion it emerges that large infrared objects such as that found by Low should characterize the late stages in the collapse of an interstellar cloud into a cluster of protostars. Because of the short lifetime of this stage, only a few highly luminous infrared gas clouds can be expected to be present in the galaxy at any instant of time. The protostars themselves could have characteristic temperatures either of several hundred degrees or several thousand degrees. If the lower temperature possibility should prove correct, then only a few such protostars are likely to be found outside absorbing gas clouds.

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Note added in proof, 23 September 1968: In their discovery paper announcing the infrared nebula, D. E. Kleinmann & F. J. Low (*Astrophys. J. Lett.* **149**, L1, 1967) have derived a more accurate surface temperature of 70 °K. This lowers the luminosity of the object significantly but does not affect the conclusions of this paper.

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